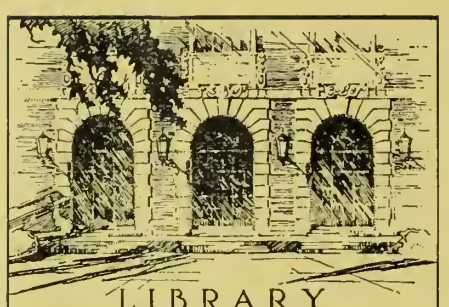

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REPORT NO. 155

TUNNEL DIODE CIRCUITS COUPLED INTO TRANSMISSION LINES

by

Tohru Moto-Oka

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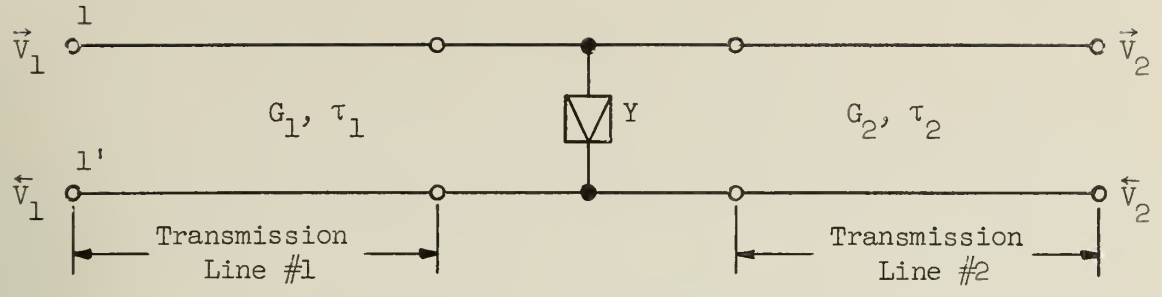
1. Introduction

All interconnecting wires should be considered as transmission lines in very high speed circuitry. Therefore, there seem to be two groups of methods which are used in the design of very high speed tunnel diode switching and logic circuits. In the first method it is assumed that all constants are ideal lumped constants and that they are interconnected ideally at first. They are then replaced by distributed circuitry. In the second method tunnel diodes are connected to distributed circuitry from the beginning. In the latter method it may be possible to discover the circuitry in which the characteristic of the distributed circuit is used in our favor. It is impossible to derive this type of circuit from the lumped constant circuit.

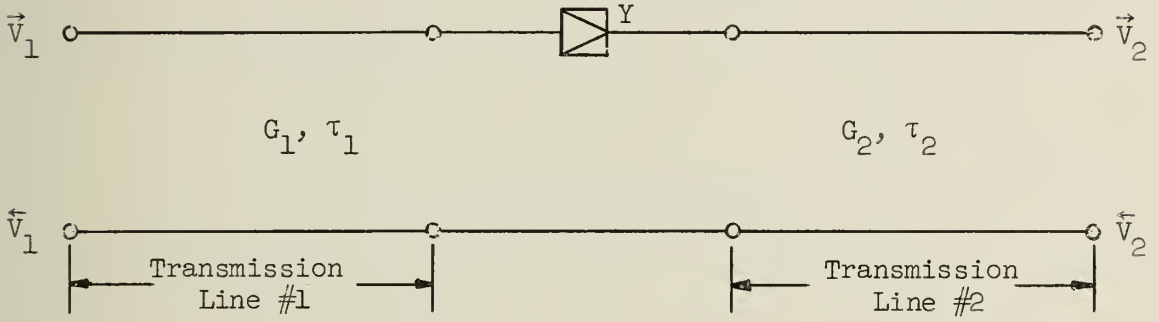
The character of the tunnel diode circuits coupled into distributed circuits is not understood well enough to design very high speed switching and logic circuits. In this report, typical tunnel diode circuits coupled into transmission lines are investigated as the preparatory stage of latter design methods.

2. Amplifiers

The circuits which are shown in Figure 1 are supposed to be the typical basic tunnel diode circuits which are coupled into transmission lines. Therefore, familiarization with the characteristics of these basic circuits is very useful in order to design more general tunnel diode circuits which are coupled to transmission lines. In addition, these circuits are expected to have the characteristics of voltage or current amplification from the investigations of tunnel diode lumped circuits, which are mentioned in an earlier report.



(a)



(b)

Figure 1. Basic Tunnel Diode Circuits Coupled Into Transmission Lines

The equations of the circuit which is shown in Figure 1(a) may be written as:

$$\begin{pmatrix} \overrightarrow{V}_2 \\ \overleftarrow{V}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2G_2}(G_1+G_2-Y)\,\epsilon^{-s(\tau_1+\tau_2)} & \frac{1}{2G_2}(-G_1+G_2-Y)\,\epsilon^{-s(\tau_2-\tau_1)} \\ \frac{1}{2G_2}(-G_1+G_2+Y)\,\epsilon^{-s(\tau_1-\tau_2)} & \frac{1}{2G_2}(G_1+G_2+Y)\,\epsilon^{-s(\tau_1+\tau_2)} \end{pmatrix} \begin{pmatrix} \overleftarrow{V}_1 \\ \overleftarrow{V}_1 \end{pmatrix} \qquad (1)$$

where the transmis ion lines are assumed to have ideal characteristics and G_i and τ_i are respectively the intrinsic admittance and the transmission time of the section "i" of the transmission lines. The tunnel diode equivalent impedance Z is assumed to be $r + sL + \frac{1}{-G + sC}$, that is,

$$Y = \frac{sC - G}{1 + (sL+r)(sC-G)} \qquad (2)$$

If the terminal 1-1' is terminated by the series circuit of a signal voltage source "E" and a resistance R_1 and the terminal 2-2' by a load resistance R_2, $\overrightarrow{V}_1$ and $\overleftarrow{V}_2$ become E_2 and zero respectively, because no reflection occurs at these terminals. Therefore, the output voltage at the terminal 2-2' may be written as:

$$v_2 = \overrightarrow{V}_2 = \frac{G_1}{G_1 + G_2} E\epsilon^{-s(\tau_1+\tau_2)} \frac{s^2LC - s(GL - rC) + (1 - G \cdot r)}{s^2LC - s\left\{GL-C\left(r + \frac{1}{G_1+G_2}\right)\right\}+\left\{1-G\left(r + \frac{1}{G_1+G_2}\right)\right\}} \qquad (3)$$

The following equations are the necessary and sufficient conditions of circuit stability:

$$\begin{cases} 1 - G\left(r + \dfrac{1}{G_1+G_2}\right) \geq 0 & (4) \\ -GL + C\left(r + \dfrac{1}{G_1+G_2}\right) \geq 0 & (5) \end{cases}$$

The following conditions are the conditions of sinusoidal oscillation:

$$\begin{cases} GL > C\left(r + \dfrac{1}{G_1+G_2}\right) & (6) \\ 4LC\left\{1-G\left(r + \dfrac{1}{G_1+G_2}\right)\right\} > \left\{GL-C\left(r + \dfrac{1}{G_1+G_2}\right)\right\}^2 & (7) \end{cases}$$

$$4LC > \left\{GL+C\left(r + \dfrac{1}{G_1+G_2}\right)\right\}^2 \qquad (7')$$

And the conditions of relaxation oscillation or switching may be written as follows:

$$\begin{cases} C\left(r + \dfrac{1}{G_1+G_2}\right) > GL & (8) \\ G\left(r + \dfrac{1}{G_1+G_2}\right) > 1 & (9) \end{cases}$$

or

$$\begin{cases} GL > C\left(r + \dfrac{1}{G_1+G_2}\right) & (10) \\ \left\{GL + C\left(r + \dfrac{1}{G_1+G_2}\right)\right\}^2 > 4LC & (11) \end{cases}$$

If the intrinsic admittances of transmission lines satisfy the following condition:

$$\frac{1}{G} = r + \frac{1}{G_1 + G_2} \tag{12}$$

the output voltage may be rewritten as:

$$v_2 = \vec{V}_2 = \frac{G_1}{G_1 + G_2} E \epsilon^{-s(\tau_1+\tau_2)} \frac{(s-\alpha)(s-\beta)}{S\left\{s - \left(\frac{G}{C} - \frac{1}{LG}\right)\right\}} \tag{13}$$

where

$$\alpha,\beta = \frac{1}{2}\left(\frac{G}{C} - \frac{r}{L}\right) \pm \sqrt{\frac{1}{4}\left(\frac{G}{C} + \frac{r}{L}\right)^2 - \frac{1}{LC}} \tag{14}$$

These equations mean that this basic circuit is considered as the basic amplifier which has an infinite voltage gain at zero frequency in the ideal case.

Also, the backward input voltage may be written as:

$$\overleftarrow{V}_1 = \frac{G_1 - G_2 - Y}{G_1 + G_2 + Y} \epsilon^{-2\tau_1 s} \frac{E}{2} \tag{15}$$

Since the input impedance may be mismatched, it becomes necessary to know how much of the available power has been amplified. For this purpose, power gain will be defined in terms of transducer gain. The available power from the source is:

$$P_{in} = G_1 E^2/4 \tag{16}$$

The output power is:

$$P_o = \left(\frac{G_1}{G_1+G_2}\right)^2 E^2 G_2 \left| \frac{(s-\alpha)(s-\beta)}{s\left\{ s - \left(\frac{G}{C} - \frac{1}{LG}\right)\right\}} \right|^2 \tag{17}$$

The transducer gain is:

$$G_{Tr} = \frac{P_o}{P_{in}} = \frac{4G_1G_2}{(G_1+G_2)^2} \left| \frac{(s-\alpha)(s-\beta)}{s\left\{ s - \left(\frac{G}{C} - \frac{1}{LG}\right)\right\}} \right|^2 \tag{18}$$

The input impedance is:

$$Z_{in} = \frac{1}{G_1} \frac{G_1 \cosh\tau_1 s + (G_2+Y)\sinh\tau_1 s}{G_1 \sinh\tau_1 s + (G_2+Y)\cosh\tau_1 s} \tag{19}$$

The cascade circuit of the basic amplifier which is shown in Figure 2 is expected to have the characteristics of an amplifier. In this circuit, the number of tunnel diodes can be selected arbitrarily. The constant current supply should keep all tunnel diodes in the negative resistance regions. The negative resistances of the tunnel diodes, the characteristic impedances of the transmission lines, and the load and source resistance must satisfy some conditions which are needed so that this circuit will have the characteristics of an amplifier. Otherwise, it may work as an oscillator or a binary cell.

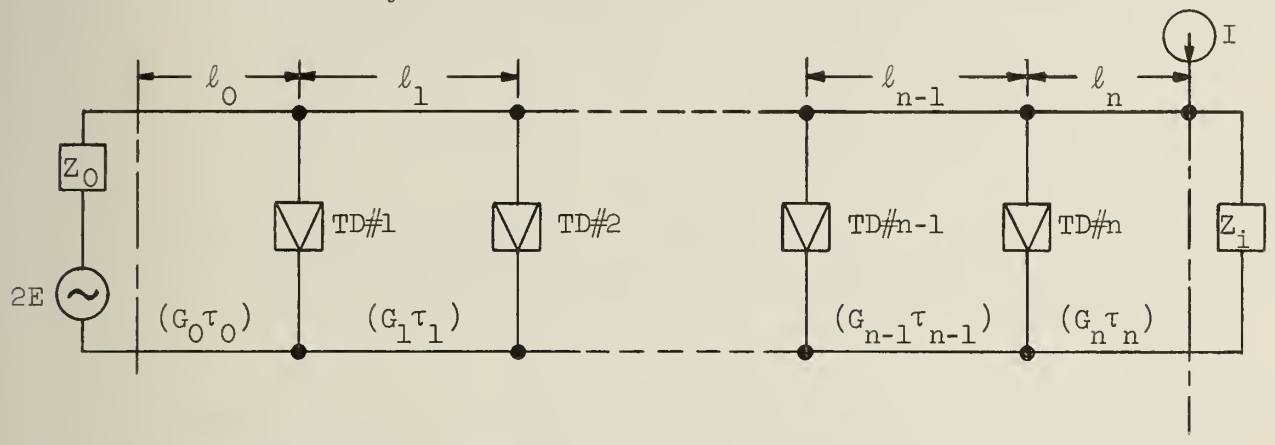


Figure 2. Transmission Line Type Amplifier

The transmission line type amplifier which has two tunnel diodes is shown in Figure 3. The circuit equations of this circuit are as follows:

$$\begin{pmatrix} \overrightarrow{V}_3 \epsilon^{s\tau_3} \\ \overleftarrow{V}_3 \epsilon^{-s\tau_3} \end{pmatrix} = \begin{pmatrix} \overrightarrow{V}_2 \\ \overleftarrow{V}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2G_3}(G_2+G_3-Y_2)\epsilon^{-s\tau_2} & \frac{1}{2G_3}(-G_2+G_3-Y_2)\epsilon^{s\tau_2} \\ \frac{1}{2G_3}(-G_2+G_3+Y_2)\epsilon^{-s\tau_2} & \frac{1}{2G_3}(G_2+G_3+Y_2)\epsilon^{s\tau_2} \end{pmatrix} \begin{pmatrix} \overrightarrow{V}_1 \\ \overleftarrow{V}_1 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} \vec{V}_1 \\ \\ \leftarrow \\ V_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2G_2}(G_1+G_2-Y_1) & \frac{1}{2G_2}(-G_1+G_2-Y_1) \\ \\ \frac{1}{2G_2}(-G_1+G_2+Y_1) & \frac{1}{2G_2}(G_1+G_2+Y_1) \end{pmatrix} \begin{pmatrix} \vec{V}_0 \, \epsilon^{-s\tau_1} \\ \\ \leftarrow \\ V_0 \, \epsilon^{s\tau_1} \end{pmatrix} \tag{21}$$

When it is assumed that the input and output terminals are terminated by matched resistors, the values of which are respectively $1/G_1$ and $1/G_3$, the voltage gain may be written as:

$$G_v = \frac{\vec{V}_3}{\vec{V}_0} = \frac{4G_1G_2\epsilon^{-s(\tau_1+\tau_3)}}{(-G_2+G_3+Y_2)(-G_1+G_2-Y_2)\,\epsilon^{-s\tau_2} + (G_1+G_2+Y_1)(G_2+G_3+Y_2)\,\epsilon^{s\tau_2}} \tag{22}$$

If it is assumed that tunnel diode admittances Y_1 and Y_2 are respectively $sC_1 - G_{T1}$ and $sC_2 - G_{T2}$, the voltage gain at a frequency ω is:

$$G'_v = \frac{2G_1G_2\epsilon^{-j\omega(\tau_1+\tau_3)}}{\left[G_2(G_1+G_3-G_{T1}-G_{T2})\cos\omega\tau_2 + \left\{\omega C_1(G_{T2}-G_3) + \omega C_2(G_{T1}-G_1)\right\}\sin\omega\tau_2\right]}$$

$$\frac{}{j\left[\omega(C_1+C_2)G_2\cos\omega\tau_2 + \left\{G_2^2 + (G_1-G_{T1})(G_3-G_{T2}) - \omega^2C_1C_2\right\}\sin\omega\tau_2\right]} \tag{23}$$

If we set $G_1 = G_{T1}$ and $G_2 = G_{T2}$, the voltage gain may be rewritten as:

$$G''_v = \frac{2G_1G_2\epsilon^{-j\omega(\tau_1+\tau_3)}}{j\left[\omega(C_1+C_2)G_2 \cos \omega\tau_2 + (G_2^2 - \omega^2C_1C_2)\sin \omega\tau_2\right]} \tag{24}$$

Also, when section 2 of the transmission line is chosen to satisfy the condition that $\tau_2G_2 = (\pi/2)\sqrt{C_1C_2}$, the voltage gain becomes infinity at $\omega = 0$ and ω_0, where ω_0 is given by $(\pi/2\tau_2)$ or $(G_2/\sqrt{C_1C_2})$. If the time constants of two tunnel diodes have the same value, that is, $C_1/G_{T1} = C_2/G_{T2} = \tau_T$, this condition may be rewritten in the following form which has clearer physical meaning:

$$\tau_2G_2 = \frac{\pi}{2}\tau_T\sqrt{G_{T1}G_{T2}} \tag{25}$$

Equation (24) may be rewritten as:

$$|\ G''_v\ | = \frac{G_1}{G_2}F(x)\ ;\ F(x) = \frac{2}{kx \cos \frac{\pi}{2}x + (1-x^2)\sin \frac{\pi}{2}x} \tag{26}$$

where

$$x = \frac{\omega}{\omega_0} \text{ and } k = \frac{C_1 + C_2}{\sqrt{C_1C_2}} \geq 2 \quad .$$

F(x) is shown in Figure 4.

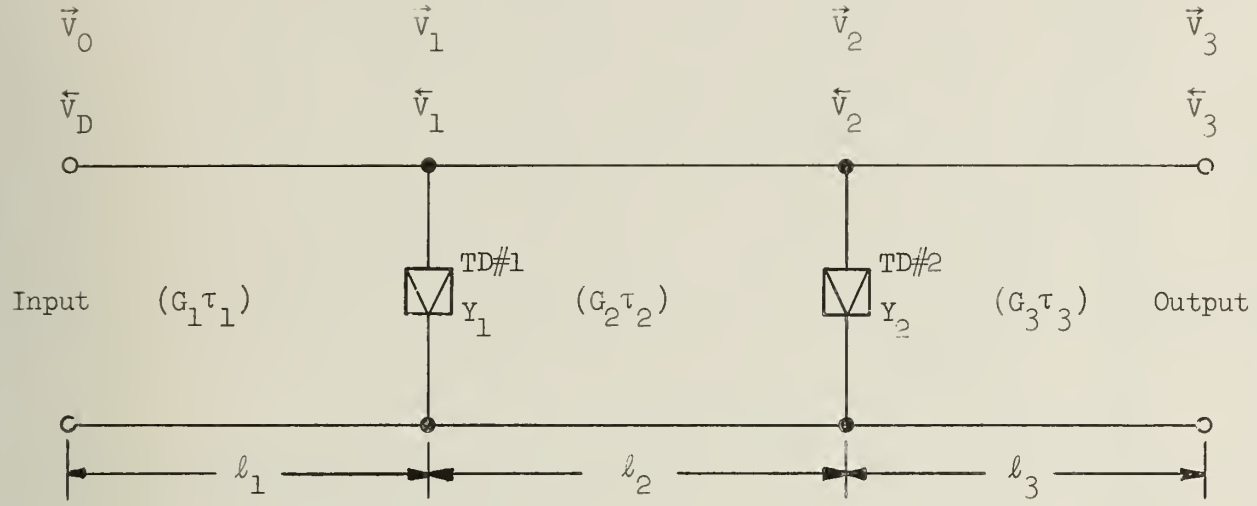


Figure 3. Transmission Line Type Amplifier with Two Tunnel Diodes

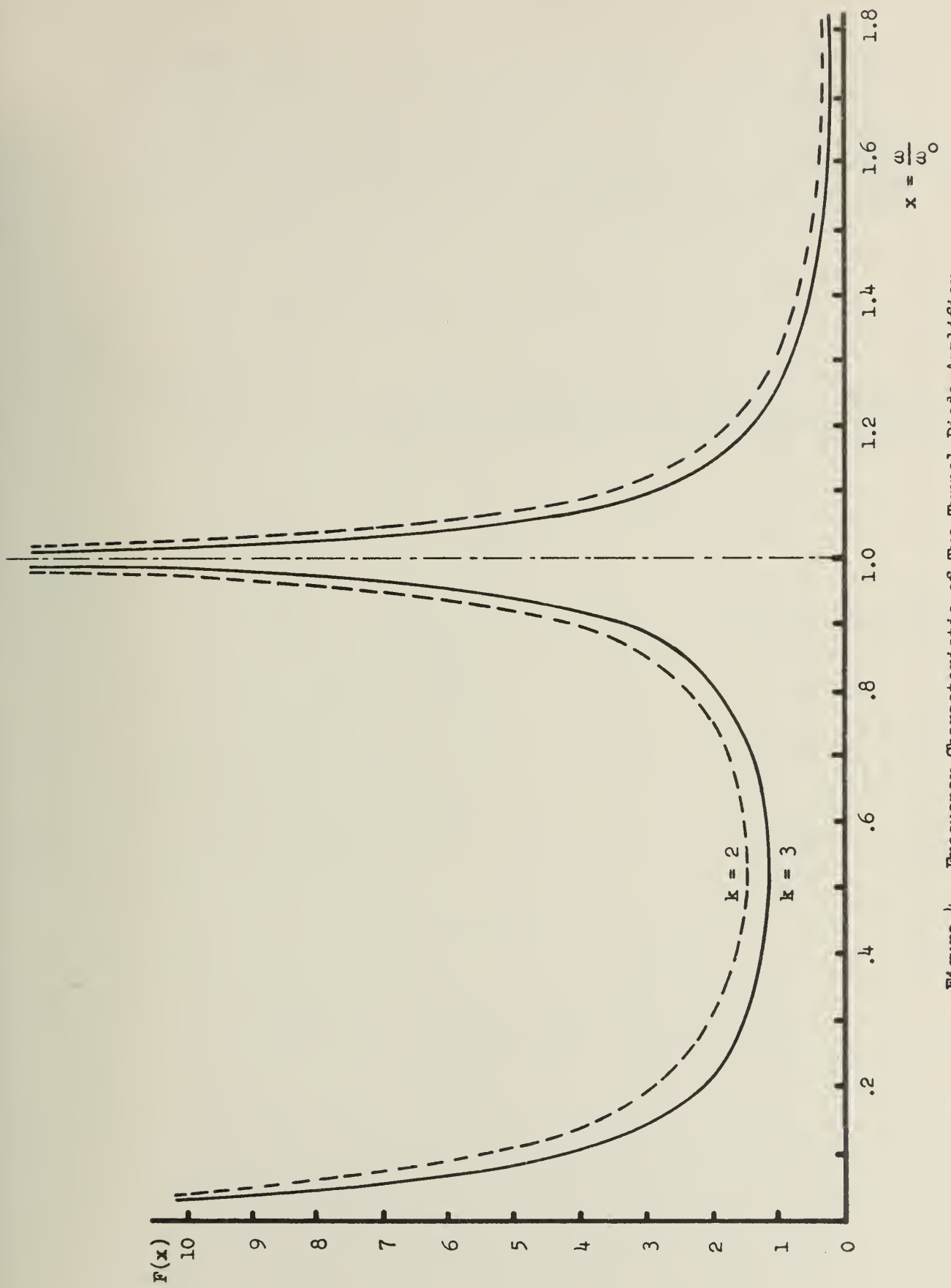


Figure 4. Frequency Characteristic of Two-Tunnel-Diode Amplifier

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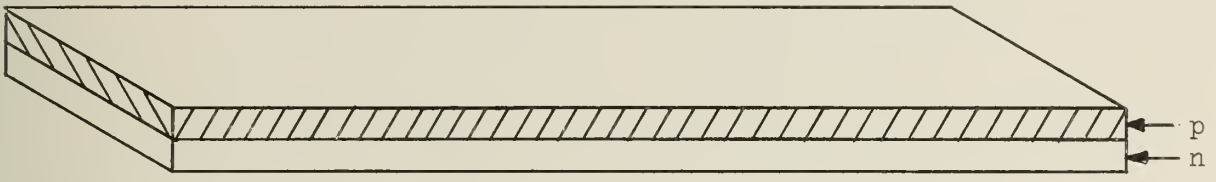
From the investigation of the two tunnel diode transmission line amplifiers, it appears to follow that amplifiers with more tunnel diodes have more peaks in the gain characteristics. It also seems that optimum gain for pulse amplification would be found with an infinite number of tunnel diodes. The ideal case is given by the strip line type tunnel diode which is shown in Figure 5.



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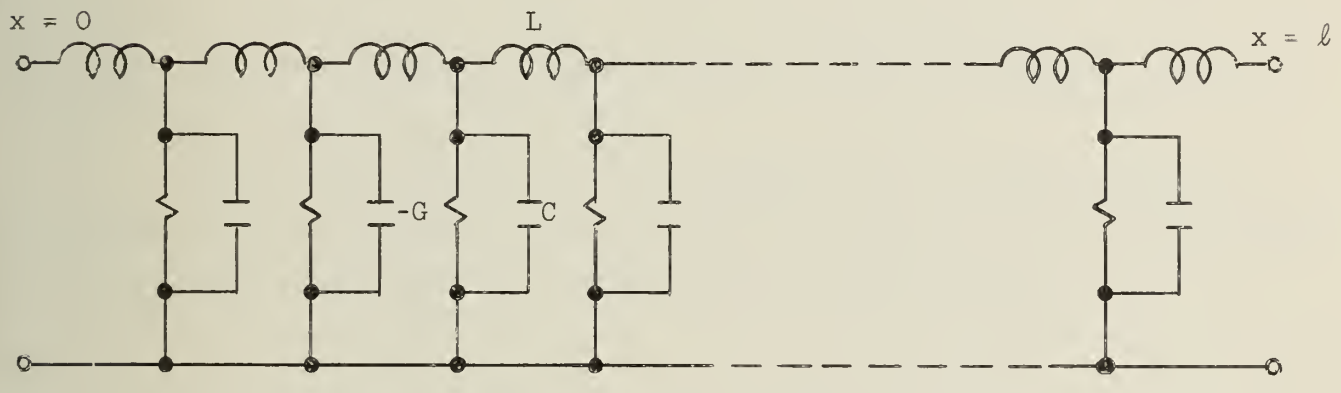


(b)

Figure 5. Strip Line Type Tunnel Diode

The characteristic equations of the strip line tunnel diode may be written as:

$$V = A\epsilon^{-\dot{\gamma}x} + B\epsilon^{-\dot{\gamma}x} \tag{27}$$

$$I = \dot{Y}_0 (A\epsilon^{-\dot{\gamma}x} - B\epsilon^{-\dot{\gamma}x} \tag{28}$$

where

$$\dot{Y}_0 = \sqrt{\frac{C}{L}\left(1-j\frac{\omega\Gamma}{\omega}\right)}\ ,\ \dot{\gamma} = \alpha + j\beta = j\omega\sqrt{LC\left(1+j\frac{\omega\Gamma}{\omega}\right)}\ ,\ \omega\Gamma = \frac{G}{C}$$

and L, C, -G are the inductance, capacitance and negative conductance per unit length of the tunnel diode respectively.

If the input terminal (x = 0) is terminated by a series circuit of a resistance R_s and a voltage source E and the output terminal (x = ℓ) is terminated by a resistance R_ℓ, then A and B are given by the following equation:

$$\begin{pmatrix} 1 + R_s\dot{Y}_0 & 1 - R_s\dot{Y}_0 \\ (1-R_\ell\dot{Y}_0)\epsilon^{-\dot{\gamma}\ell} & (1+R_\ell\dot{Y}_0)\epsilon^{\dot{\gamma}\ell} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} E \\ 0 \end{pmatrix} \tag{29}$$

The coefficients A and B are derived from this equation:

$$A = \frac{(1+R_\ell \dot{Y}_0)\epsilon^{\dot{\gamma}\ell}}{(R_s+R_\ell)\dot{Y}_0 \cosh \dot{\gamma}\ell + (1+R_s R_\ell \dot{Y}_0^2)\sinh \dot{\gamma}\ell} \frac{E}{2} \tag{30}$$

$$B = \frac{(R_\ell \dot{Y}_0 - 1)\epsilon^{-\dot{\gamma}\ell}}{(R_s+R_\ell)\dot{Y}_0 \cosh \dot{\gamma}\ell + (1+R_s R_\ell \dot{Y}_0^2)\sinh \dot{\gamma}\ell} \tag{31}$$

The output voltage which is the voltage at $x = \ell$ may be written as:

$$v_\ell = \frac{E R_\ell \dot{Y}_0}{(R_s+R_\ell)\dot{Y}_0 \cosh \dot{\gamma}\ell + (1+R_s R_\ell \dot{Y}_0^2)\sinh \dot{\gamma}\ell}$$

The voltage gain which is defined by the ratio of the output voltage and the source voltage is derived from the above equation.

$$G_v = \frac{v}{E} = \frac{R_\ell}{R_s + R_\ell} \frac{1}{\cosh \dot{\gamma}\ell + \left\{\frac{1+R_s R_\ell \dot{Y}_0^2}{(R_s+R_\ell)\dot{Y}_0}\right\} \sinh \dot{\gamma}\ell} \tag{32}$$

and

$$| G_v | = \frac{2R_\ell}{R_s + R_\ell} \frac{1}{\sqrt{(1+N^2+M^2)^2 - 4M^2}} \frac{1}{\sqrt{\cosh(2\alpha\ell+\varphi_1) - \cos(2\beta\ell-\varphi_2)}} \tag{33}$$

where

$$M + jN = \frac{1 + R_s R_\ell \dot{Y}_0^2}{(R_s+R_\ell)\dot{Y}_0} = \frac{1 + R_s R_\ell \left(\frac{C}{L}\right) \left(1+j\frac{\omega_T}{\omega}\right)}{(R_s+R_\ell)\sqrt{\left(\frac{C}{L}\right)\left(1+j\frac{\omega_T}{\omega}\right)}}$$

$$\varphi_1 = \tanh^{-1} \frac{2M}{1 + M^2 + N^2} \quad , \text{ and}$$

$$\varphi_2 = \tan^{-1} \frac{2N}{M^2 + N^2 - 1}$$

The approximated equation of the gain at low frequencies is derived from Eq. (32) as:

$$G_v \doteq \frac{R_\ell}{R_s + R_\ell} \frac{1}{1 + (\dot{\gamma}\ell)^2 + \frac{1 + R_s R_\ell Y_0^2}{(R_s+R_\ell)\dot{Y}_0}(\dot{\gamma}\ell)} \tag{34}$$

$$= \frac{R_\ell}{R_s + R_\ell} \frac{1}{\left(1 - \frac{R_s R_\ell}{R_s+R_\ell} G\ell - \frac{1}{2}\omega^2 LC\ell^2\right) + j\omega L\ell \left(\frac{1+R_s R_\ell \frac{C}{L}}{R_s+R_\ell} - \frac{G\ell}{2}\right)} \tag{34'}$$

When $G\ell = (1/R_s) + (1/R_\ell)$, the voltage gain becomes infinity at zero frequency. If

$$G\ell = 2\,\frac{1 + R_s R_\ell \frac{C}{L}}{R_s + R_\ell}$$

then the voltage gain in the low frequency region may be rewritten as:

$$G_v = \frac{R_\ell}{R_s + R_\ell}\;\frac{1}{1 - \frac{R_s R_\ell}{R_s + R_\ell} G\ell - \frac{1}{2}\omega^2 LC\ell^2 + j\omega^3 P(\omega)} \tag{35}$$

where $P(\omega)$ is a polynomial in $j\omega$. If these two conditions are satisfied concurrently, that is:

$$G\ell = \frac{1}{R_s} + \frac{1}{R_\ell} \text{ and } \frac{C}{L} = Y_0 = \frac{1}{2}\left(\frac{1}{R_s^2} + \frac{1}{R_\ell^2}\right) \tag{36}$$

the frequency characteristic of the voltage gain at low frequencies becomes

$$G_v \doteq -\frac{2R_\ell}{R_s + R_\ell}\,\frac{1}{\omega^2 LC\ell^2} \tag{37}$$

When it is assumed that $R_s = R_\ell = 2/(G\ell) = \sqrt{L/C}$, which satisfies Eq. (36), the absolute value of the voltage gain is derived from Eq. 33 as:

$$|G_v| = \frac{4\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2}}{\left(\frac{\omega_T}{\omega}\right)^2} \quad \frac{1}{\sqrt{\cosh(2\alpha\ell+\varphi_1) - \cos(2\beta\ell-\varphi_2)}} \tag{38}$$

where

$$\varphi_1 = 2\tanh^{-1}\sqrt{\frac{1}{2}\left\{1+\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2}\right\}}$$

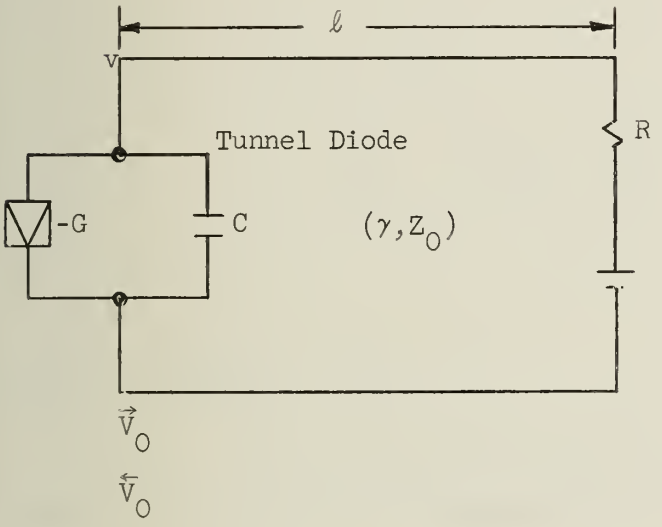
$$\varphi_2 = 2\tan^{-1}\sqrt{\frac{1}{2}\left\{\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2} - 1\right\}}$$

$$\alpha = -\omega\sqrt{LC}\sqrt{\frac{1}{2}\left\{\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2} - 1\right\}} = \omega\sqrt{LC}\tan\frac{\varphi_2}{2}$$

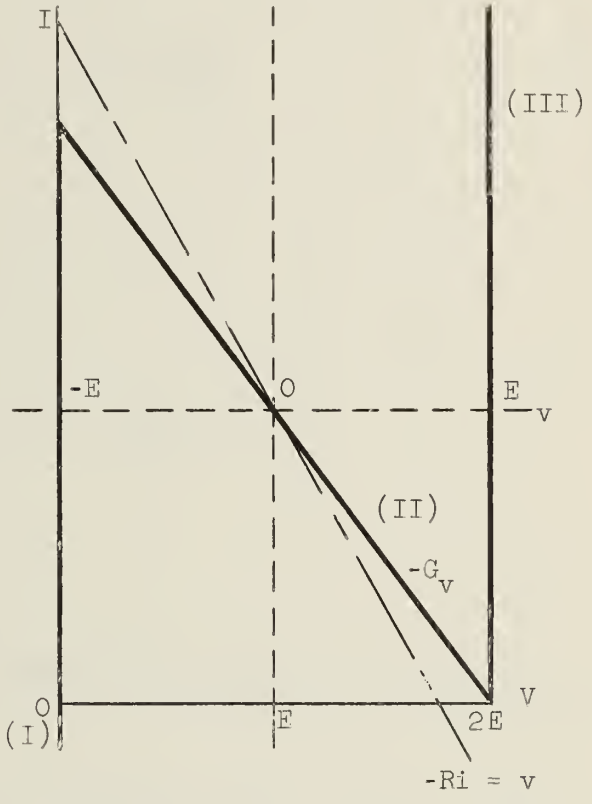
$$\beta = \omega\sqrt{LC}\sqrt{\frac{1}{2}\left\{\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2} + 1\right\}} = \omega\sqrt{LC}\tanh\frac{\varphi_1}{2}$$

3. Oscillator

It is apparent from the stability criterion of the tunnel diode amplifier which was mentioned in Eq. (4) and Eq. (5) that the tunnel diode circuit coupled into a transmission line may oscillate under some conditions. The behavior of this type of the tunnel diode oscillator which is shown in Figure 6 is analyzed from two different standpoints, that is, the small signal linear analysis and the large signal analysis which uses the broken line approximation of the tunnel diode characteristic.



(a) Equivalent Circuit



(b) Broken Line Approximation of the Tunnel Diode Characteristic

Figure 6. The Tunnel Diode Oscillator

The equivalent circuit and the broken line approximation of the tunnel diode characteristic which are shown in Figure 6 are used in the large signal analysis of the circuit behavior of the tunnel diode oscillator. The circuit equations of the oscillator may be written as:

$$v = \overrightarrow{V}_0 + \overleftarrow{V}_0 \tag{39}$$

$$\overleftarrow{V}_0 = \frac{R - Z_0}{R + Z_0} \epsilon^{-2st_0} \overrightarrow{V}_0 = \rho\epsilon^{-2st_0} \overrightarrow{V}_0 \tag{40}$$

$$\left.\begin{array}{ll} v = -E & \text{in the region I} \\ \dfrac{dv}{d\tau} + (1-GZ_0)v = 2\overleftarrow{V}_0 & \text{in the region II} \\ v = E & \text{in the region III} \end{array}\right\} \tag{42}$$

where $\tau = \dfrac{t}{Z_0 C}$ m, $t_0 = \sqrt{\mu\epsilon}\ell$.

If it is assumed that the switching time of the tunnel diode is negligible compared to the transmission time t_0, the following points become apparent:

1) It is assumed that the tunnel diode is switched from v = -E to v = E at t = 0.
2) During $0 < t < 2t_0$, $\overleftarrow{V}_0$ is assumed to be V_1, which should be larger than $(1/2)(1-GZ_0)E$ in order that the tunnel diode remains in region III, that is, v = E.

3) During $2t_0 < t < 4t_0$, $\overleftarrow{V}_0 = \rho(E-V_1)$ and the tunnel diode remains in region III if $2\rho(E-V_1) + (GZ_0-1)E > 0$. Otherwise, the tunnel diode is switched back to region I at $t = 2t_0$.

4) During $4t_0 < t < 6t_0$,

a) if the tunnel diode remained in region III during $2t_0 < t < 4t_0$, $\overleftarrow{V}_0 = \rho\left\{E - \rho(E-V_1)\right\}$ then the diode remains in the region III if $2\rho\left\{E - \rho(E-V_1)\right\} + (GZ_0-1)E \geq 0$. Otherwise, it is switched back to region I at $t = 4t_0$.

b) if the tunnel diode is switched back to region I at $t = 2t_0$, $\overleftarrow{V}_0 = \rho\left\{-E - \rho(E-V_1)\right\}$ the tunnel diode will remain in the region I if $2\rho\left\{-E - \rho(E-V_1)\right\} - (GZ_0-1)E \leq 0$. Otherwise, it is again switched back to region III at $t = 4t_0$.

In general, if the tunnel diode stays in region III during $0 < t < 2(m-1)t_0$, the backward voltage may be written as:

$$\overleftarrow{V}_0 = (-\rho)^{m-1} V_1 - \sum_{n=1}^{m-1} (-\rho)^n E \qquad (42)$$

during $2(m-1)t_0 < t < 2mt_0$. The tunnel diode will remain in region III if the following condition is satisfied:

$$2\overleftarrow{V}_0 + (GZ_0-1)E \geq 0 \qquad (43)$$

Otherwise it is switched back to the region I at $t = 2(m-1)t_0$.

In other words, if the following two conditions are satisfied,

$$-E\sum_{n=1}^{m-1}(-\rho)^n + (-\rho)^{m-1}V_1 = \frac{\rho E}{1+\rho}\left\{1-(-\rho)^{m-1}\right\} + (-\rho)^{m-1}V_1 \geq \frac{-E}{2}(GZ_0-1) \quad (44)$$

$$-E\sum_{n=1}^{m}(-\rho)^n + (-\rho)^{m}V_1 = \frac{\rho E}{1+\rho}\left\{1-(-\rho)^{m}\right\} + (-\rho)^{m}V_1 < \frac{-E}{2}(GZ_0-1) \quad (45)$$

the tunnel diode is switched from E to -E at $t = 2mt_0$. This behavior is illustrated in Figure 7.

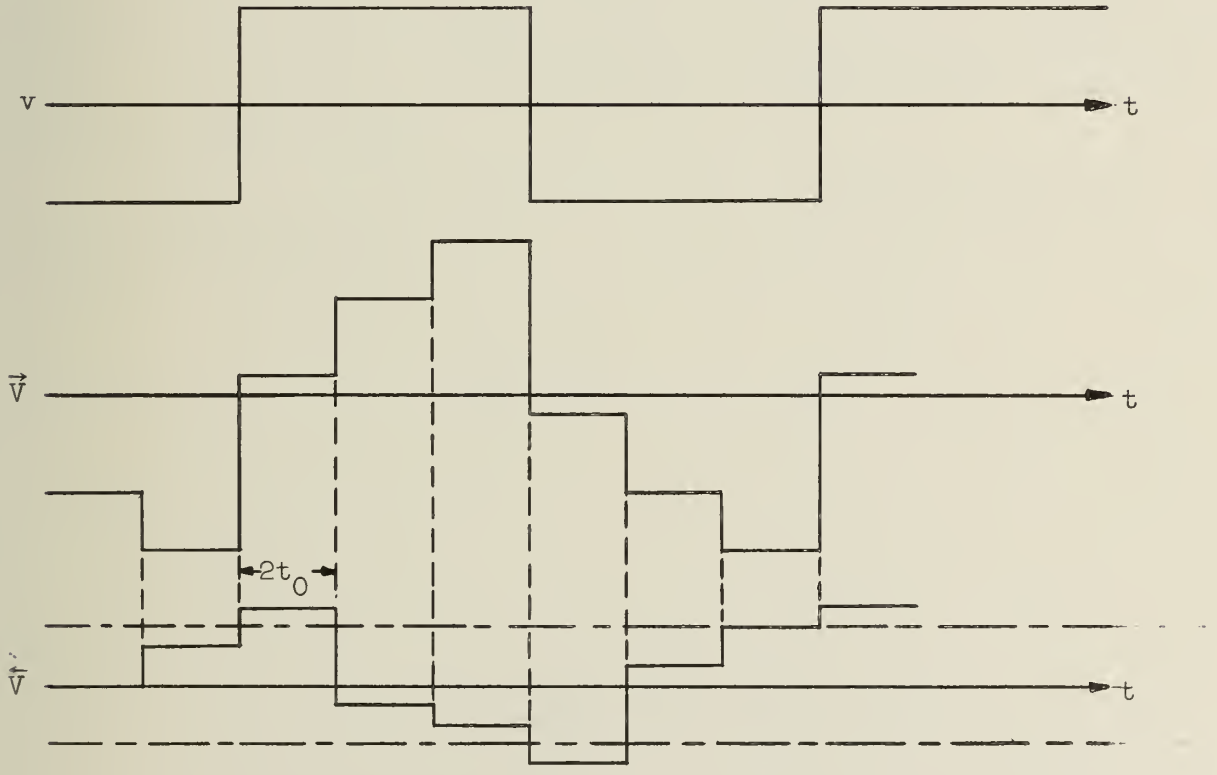


Figure 7. The Waveform of Ideal Oscillation

The steady state oscillation condition may be written as:

$$-V_1 = -E \sum_{n=1}^{m} (-\rho)^n + (-\rho)^m V_1 \tag{46}$$

or

$$V_1 = \frac{\rho E}{1 + \rho} \frac{1 - (-\rho)^m}{1 + (-\rho)^m} \tag{46'}$$

It is worthwhile to mention that in the above discussion V_1 must be larger than $(1/2)(GZ_0-1)E$ in order to switch the tunnel diode from -E to E. And also, GZ_0 must be larger than 1 for switching behavior. This condition is apparent from Eq. (41). It is apparent from Eq. (46') that ρ should be negative, that is, R should be smaller than Z_0.

"m" must be one in order to get oscillation of maximum frequency. This condition may be written as:

$$\frac{-2\rho}{1 - \rho} > GZ_0 - 1 > \frac{2\rho}{1 - \rho} \tag{47}$$

or

$$1 - \frac{R}{Z_0} > GZ_0 - 1 > \frac{R}{Z_0} - 1 \tag{47'}$$

This condition means that ρ should be negative, but GZ_0 may be larger or smaller than one. The latter part means that even if switching behavior is not assumed, oscillation of a tunnel diode is possible. Z_0 should be in the following region for the relaxation oscillation of frequency of approximately $(1/4t_0)$:

$$1 < GZ_0 < 1 + \sqrt{1 - GR} \tag{48}$$

If V is given by $-\rho E/(1-\rho)$, the switching time of the tunnel diode is derived from the following equation:

$$\frac{dv}{d\tau} = av - \frac{2\rho E}{1 - \rho} \tag{49}$$

where $a = GZ_0 - 1$, $\tau = (t/Z_0C)$.

At $\tau = 0$, $v = -E$, then

$$v = \left\{ \frac{-2\rho}{a(1-\rho)} (\epsilon^{a\tau} - 1) - \epsilon^{a\tau} \right\} E \tag{50}$$

At $\tau = \tau_1$, $v = E$, then

$$a\tau_1 = \log \frac{-2\rho + a(1-\rho)}{-2\rho - a(1-\rho)} \tag{51}$$

Then the maximum frequency of relaxation oscillation is expected to be in the following region:

$$\frac{1}{2Z_0C\tau_1} > f_m > \frac{1}{4Z_0C\tau_1} \tag{52}$$

or

$$\frac{a}{2Z_0C \log \frac{-2\rho + a(1-\rho)}{-2\rho - a(1-\rho)}} > f_m > \frac{a}{4Z_0C \log \frac{-2\rho + a(1-\rho)}{-2\rho - a(1-\rho)}} \tag{52'}$$

Equation (51) may be rewritten as:

$$a\tau_1 = \log \frac{x+1}{x-1} = 2\left[\frac{1}{x} + \frac{1}{3}\,\frac{1}{x^3} + \frac{1}{5}\left(\frac{1}{x}\right)^5 \cdots\right] \tag{51'}$$

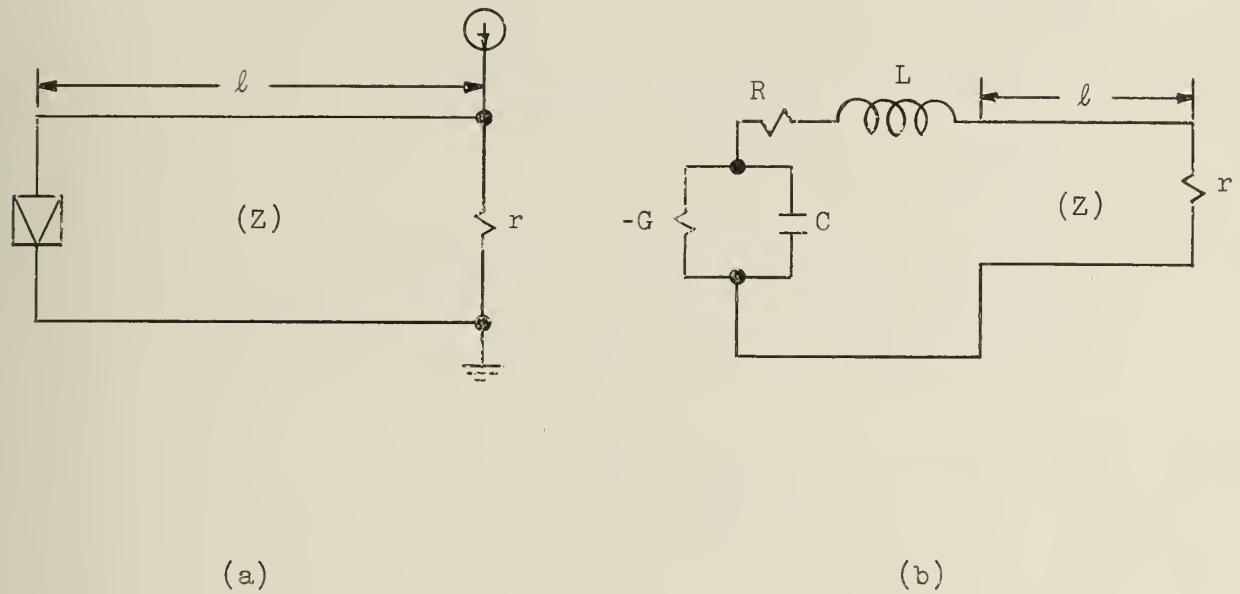
where

$$x = \frac{-2\rho}{a(1-\rho)} = \frac{Z_0 - R}{(GZ_0-1)Z_0} > 1$$

Therefore,

$$f_m < \frac{1}{2Z_0C\tau_1} < \frac{1}{4Z_0C}\left(1 - \frac{R}{Z_0}\right) < \frac{1}{4Z_0C} \tag{53}$$

The above discussion gives some ideas on the oscillation conditions of the tunnel diode coupled into a transmission line. Another method of approach to the oscillation conditions is a small signal linear analysis. The equivalent circuit for small signal operation of the tunnel diode oscillator shown in Figure 8(a) is given in Figure 8(b).



(a) (b)

Figure 8. The Equivalent Circuit of the Tunnel Diode Oscillator for Small Signal Operation

The oscillating conditions of this equivalent circuit may be written as:

$$G = \mathrm{Re}\left(\frac{1}{R+j\omega L+Z\,\dfrac{1+\rho\epsilon^{-j2\omega t_0}}{1-\rho\epsilon^{-j2\omega t_0}}} \right) \tag{54}$$

$$-\omega C = \mathrm{Im}\left(\frac{1}{R+j\omega L+Z\,\dfrac{1+\rho\epsilon^{-j2\omega t_0}}{1-\rho\epsilon^{-j2\omega t_0}}} \right) \tag{55}$$

where $\rho = \dfrac{r - Z}{r + Z}$ and $t_0 = \sqrt{\mu\epsilon}\,\ell$.

These equations may be rewritten as follows:

$$\begin{cases} G = \dfrac{R(1+\rho^2-2\rho\cos 2\omega t_0) + (1-\rho^2)Z}{(R^2+\omega^2L^2)(1+\rho^2-2\rho\cos 2\omega t_0)+ 2\left\{R(1-\rho^2)- 2\rho\omega L\sin 2\omega t_0\right\} Z} \cdot \\ \qquad \dfrac{1}{(1+\rho^2+2\rho\cos 2\omega t_0)Z^2} \end{cases} \tag{54'}$$

$$\begin{cases} \omega C = \dfrac{\omega L(1+\rho^2-2\rho\cos 2\omega t_0) -2\rho\sin 2\omega t_0\, Z}{(R^2+\omega^2L^2)(1+\rho^2-2\rho\cos 2\omega t_0)+ 2\left\{R(1-\rho^2) -2\rho\omega L\sin 2\omega t_0\right\} Z} \cdot \\ \qquad \dfrac{1}{(1+\rho^2+2\rho\cos 2\omega t_0)Z^2} \end{cases} \tag{55'}$$

or

$$\begin{cases} \left[G\left\{(R^2+\omega^2L^2+Z^2)(1+\rho^2)+ 2ZR(1-\rho^2)\right\} - R(1+\rho^2) - (1-\rho^2)Z\right] \\ -2\rho\cos 2\omega t_0\left\{G(R^2+\omega^2L^2-Z^2)- R\right\} -2\rho\sin 2\omega t_0(2\omega LGZ) = 0 \end{cases} \tag{54''}$$

$$\begin{cases} \omega C\left\{(R^2+\omega^2L^2 - \frac{L}{C} + Z^2)(1+\rho^2)+ 2ZR(1-\rho^2)\right\} \\ -2\rho\cos 2\omega t_0\left\{\omega C(R^2+\omega^2L^2 - \frac{L}{C} - Z^2)\right\} -2\rho\sin 2\omega t_0(2\omega^2LC-1)Z = 0 \end{cases} \tag{55''}$$

or

$$(1+\rho^2)\left\{G(R^2+\omega^2L^2+Z^2)-R+\left(\frac{1-\rho^2}{1+\rho^2}\right)(2GR-1)Z\right\}$$

$$-2\rho\sqrt{\left\{G(R^2+\omega^2L^2-Z^2)-R\right\}^2+4(\omega LGZ)^2}\,\sin(2\omega t_0+\alpha)=0 \qquad (54''')$$

where

$$\alpha = \text{Arc tan}\,\frac{G(R^2+\omega^2L^2-Z^2)-R}{2\omega LGZ}$$

$$(1+\rho^2)\omega C\left\{R^2+\omega^2L^2-\frac{L}{C}+Z^2+\left(\frac{1-\rho^2}{1+\rho^2}\right)(2ZR)\right\}$$

$$-2\rho\sqrt{\left\{\omega C(R^2+\omega^2L^2-\frac{L}{C}-Z^2)\right\}^2+(2\omega^2LC-1)^2Z^2}\,\sin(2\omega t_0+\beta)=0 \qquad (55''')$$

where

$$\beta = \text{Arc tan}\,\frac{\omega C\left(\omega^2L^2+R^2-\frac{L}{C}-Z^2\right)}{(2\omega^2LC-1)Z}$$

The relation between oscillation frequency and the length of the transmission line can be derived from Eq. (55), if the other circuit constants are given, because this equation does not include G. The negative conductance value is given by Eq. (54) after inserting the oscillating frequency and the traveling time of the transmission line, which in turn are derived from Eq. (55) and Eq. (54). This negative conductance seems to be the average negative conductance occurring in the large signal operation of the device. Some information about the oscillation amplitude can be obtained in this way.

Examples of the relationship between ω and t_0 and the relationship between G and t_0 are given in Figure 9. The dependence of G on ω, which is derived from Figure 9, is shown also in Figure 10. In this case, it is assumed that C = 4pF, R = 1.5Ω, L = 0.6nH and Z = 50Ω. It is worthwhile to mention that the difference $n\pi/\omega$ of the transmission time does not change anything and that the relation of Figure 9 corresponds to the case for n = 0.

The upper frequency limit of oscillation is derived easily from Eq. (55'''). If the coefficient of the sine function is larger than the first term of Eq. (55'''), any t_0 does not satisfy this equation, that is, the upper frequency limit is derived from the following equation:

$$\omega_m^2 C^2 \left\{R^2 + \omega_m^2 L^2 - \frac{L}{C} + Z^2 + \left(\frac{1-\rho^2}{1+\rho^2}\right)2ZR\right\}^2$$

$$= \frac{4\rho^2}{(1+\rho^2)^2}\left\{\omega_m^2 C^2\left(R^2+\omega_m^2 L^2 - \frac{L}{C} - Z^2\right)^2 + (2\omega_m^2 LC-1)^2 Z^2\right\} \qquad (56)$$

At the upper limit of the oscillation frequency, the length of the transmission line must be

$$\ell_{\omega=\omega_m} = \frac{t_0}{\sqrt{\mu\epsilon}}\Bigg|_{\omega=\omega_m} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{2\omega_m}\left\{\frac{3}{2}\pi + 2n\pi - \text{Arc tan}\frac{\omega_m C\left(R^2+\omega_m^2 L^2 - \frac{C}{L} - Z^2\right)}{(2\omega_m^2 LC-1)Z}\right\} \qquad (57)$$

The negative conductance may be written as:

$$G_{\omega=\omega_m} = \frac{R(1+\rho^2+2\rho\sin\beta_m) + (1-\rho^2)Z}{(R^2+\omega_m^2 L^2)(1+\rho^2+2\rho\sin\beta_m) + 2\left\{R(1-\rho^2) + 2\rho\omega_m L\cos\beta_m\right\}Z + (1+\rho^2-2\rho\sin\beta_m)Z^2}$$

where

$$\beta_m = \text{Arc tan}\frac{\omega_m C(\omega_m^2 L^2+R^2 - \frac{L}{C} - Z^2)}{(2\omega_m^2 LC-1)Z} \qquad (58)$$

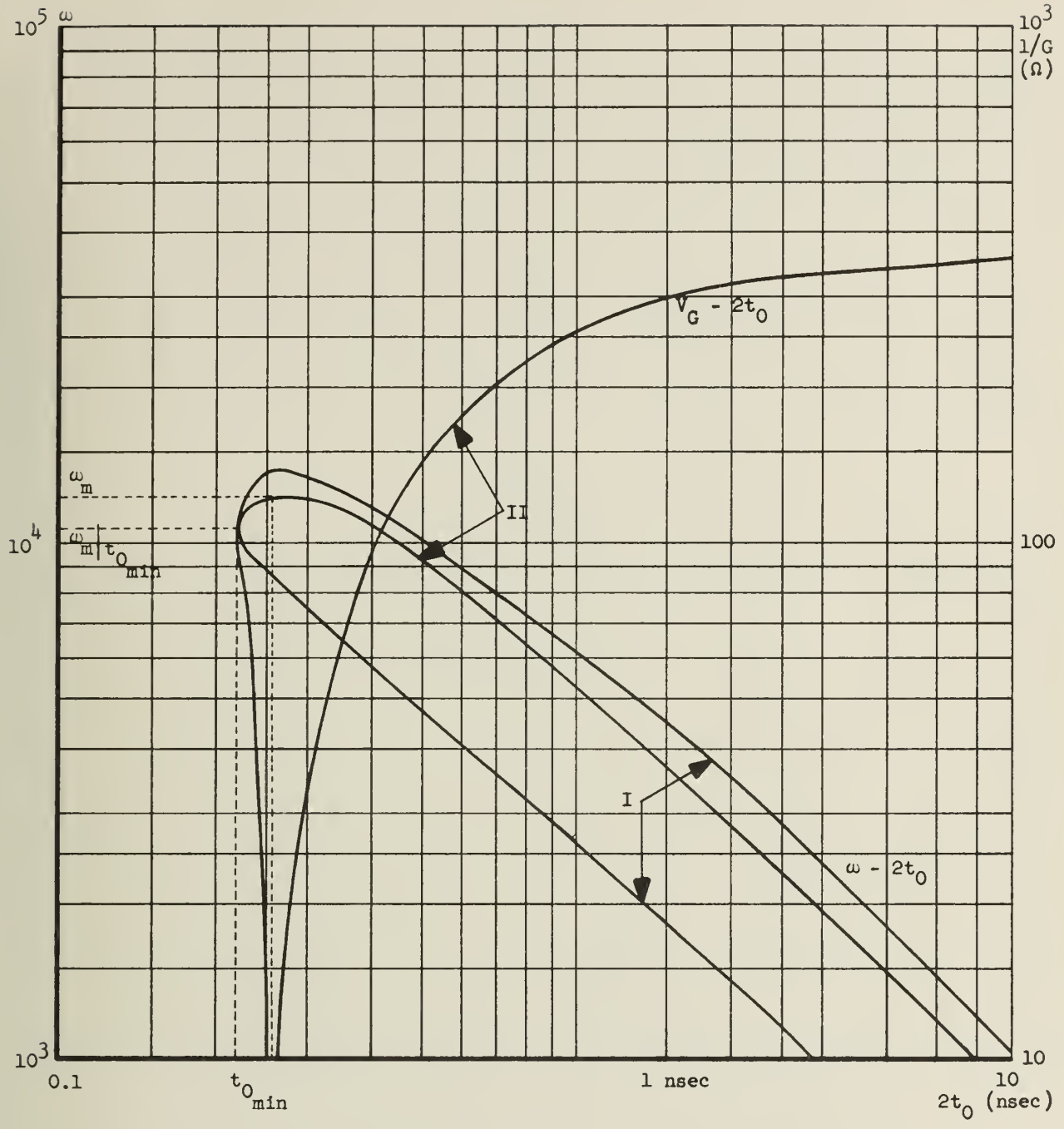


Figure 9. The Relation Between t_0 and ω or G of the Oscillator

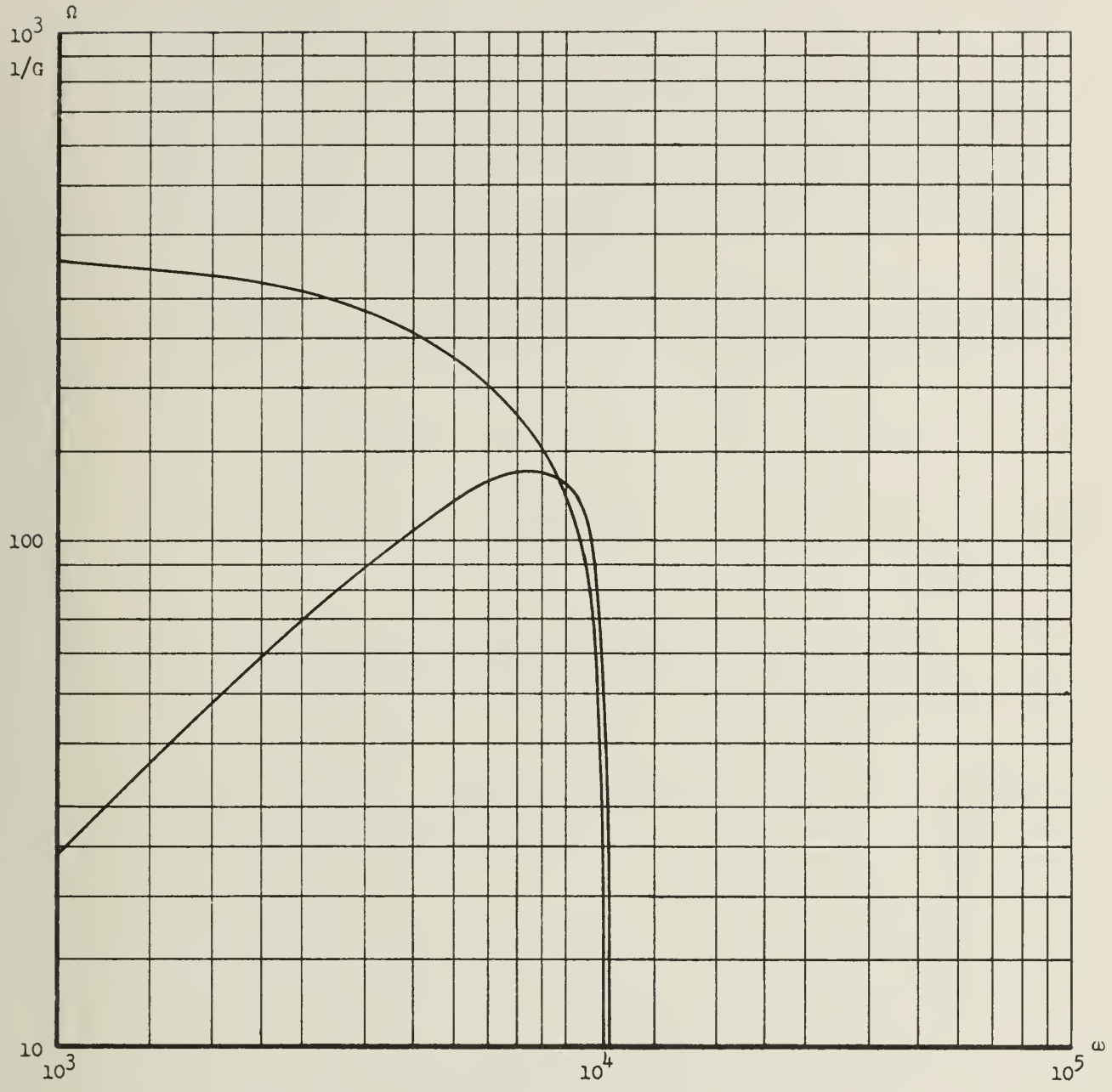


Figure 10. The Relation Between Oscillation Frequency and Negative Conductance of the Oscillator

It is evident from Figure 9 or Figure 10 that the negative conductance at the upper frequency limit of oscillation becomes very large, so that the maximum oscillation frequency seems to be limited, in fact, by the maximum value of the negative conductance of the tunnel diode. Since Figure 9 shows that two principal oscillation modes correspond to a given length of transmission line, the maximum oscillation frequency seems to correspond to the minimum length of a transmission line when the oscillation mode which corresponds to the smaller value of G is predominant. These values are shown in Figure 9.

If ρ is assumed to be -1, the relations mentioned above are simplified. The upper limit of the oscillation frequency and the negative conductance may be rewritten as:

$$\omega'_m = \frac{1}{2RC} \tag{59}$$

$$G_{\omega=\omega'_m} = \frac{R(1-\sin\beta'_m)}{(R^2+\omega'^2_m L^2+Z^2) - (R^2+\omega'^2_m L^2-Z^2)\sin\beta'_m - 2\omega'_m LZ\cos\beta'_m}$$

$$= \frac{R}{R^2 + \left(\omega'_m L \pm \sqrt{\frac{1+\sin\beta'_m}{1-\sin\beta'_m}}\, Z\right)^2} = \frac{1}{2R} \tag{60}$$

From Eq. (59) and Eq. (60):

$$\omega'_m = \frac{G_{\omega=\omega'_m}}{C} < \frac{G_0}{C} \tag{61}$$

where G_0 is the maximum absolute value of the negative conductance of the tunnel diode.

Therefore, the upper frequency limit in this case is given by

$$\omega'_m = \text{Min.}(G_0/C,\ 1/2RC) \tag{62}$$

If it is assumed that the series impedance of the tunnel diode, that is, R + sL can be neglected, the oscillation conditions may be rewritten as follows:

$$\begin{cases} G = \dfrac{(1-\rho^2)}{(1+\rho^2+2\rho\cos 2\omega t_0)Z} & (63) \\ \omega C = \dfrac{-2\rho \sin 2\omega t_0}{(1+\rho^2+2\rho\cos 2\omega t_0)Z} & (64) \end{cases}$$

These conditions are illustrated more explicitly by Figure 11 where GZ is given by $(a/b)^2$ and ωCZ by c/b^2. The maximum value of ωCZ may be written as:

$$\omega CZ\,|_{Max} = \frac{-2\rho}{1-\rho^2} \tag{65}$$

In this case,

$$\cos 2\omega_m t_0 = \frac{-2\rho}{1+\rho^2} \tag{66}$$

$$ZG_{\omega=\omega_m} = \frac{1+\rho^2}{1-\rho^2} \tag{67}$$

The upper limit of the oscillation frequency is derived from Eq. (65) and Eq. (67) as:

$$\omega_m = \frac{-2\rho}{1+\rho^2}\frac{G}{C} \tag{68}$$

Also, Eq. (53) may be rewritten as:

$$\omega_m < \frac{\pi}{2}\frac{1}{Z_0 C} < \frac{\pi}{2}\frac{G}{C} \tag{53'}$$

Therefore, it is concluded from a comparison of Eq. (53'), (62), and (68) that the upper limit of the oscillation frequency is given as $G_0/(2\pi C)$ in an ordinary tunnel diode in which R is smaller than $1/G_0$.

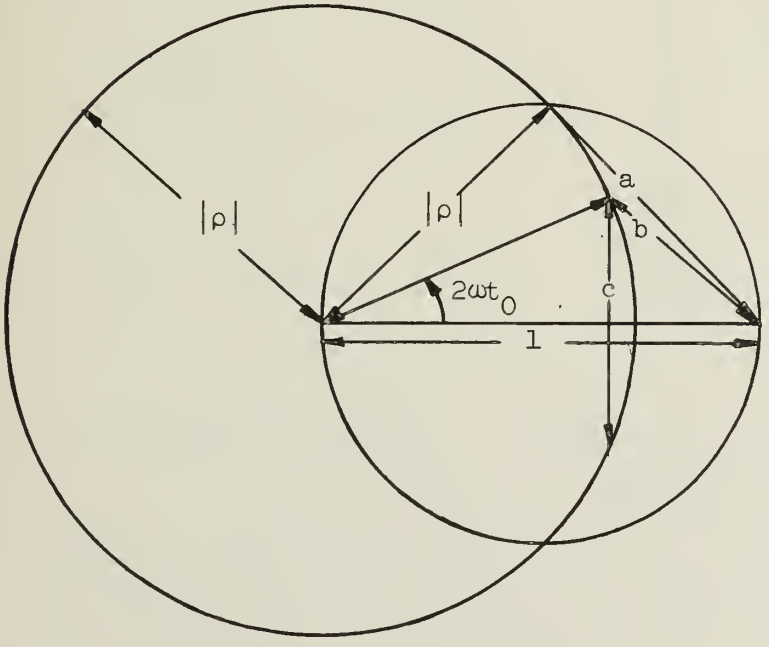


Figure 11. Illustration of the Oscillation Condition

4. Monostable Multivibrator

The circuit which is shown in Figure 12 works as a monostable multivibrator when some conditions are satisfied. The optimum design conditions based on the assumption that the characteristic curve of the tunnel diode is approximated by the idealized N-shape split line, which is shown in Figure 13, are mentioned below. The other assumptions are that the equivalent circuit of the tunnel diode is given by the parallel circuit of a constant capacitance and the nonlinear conductance. The conductance is expressed as $i = f(v)$. Furthermore, it is assumed that the trigger input is the constant current pulse $\bar{I}_i$, its width is longer than the rise time (t_r) of the output waveform and shorter than twice the transmission time (t_0) of the transmission line, that is:

$$\bar{I}_i = I_i(1-\epsilon^{-st_i}) \quad , \quad t_r \leq t_i \leq 2t_0 \tag{69}$$

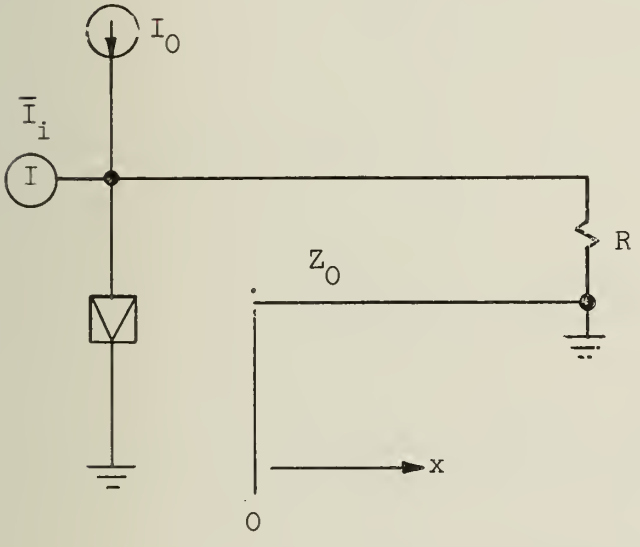


Figure 12. Monostable Multivibrator

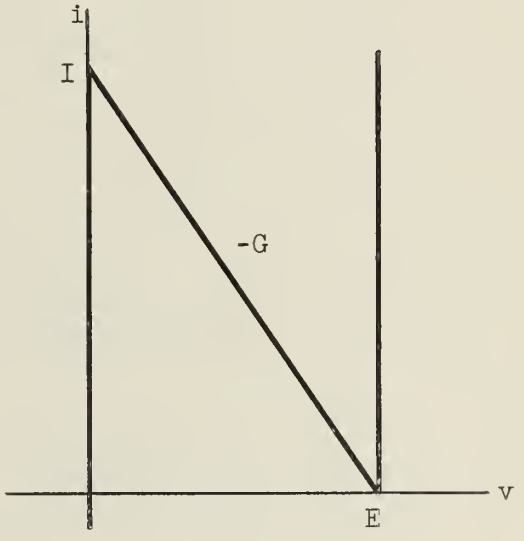


Figure 13. The N-Shape Approximation of the Tunnel Diode Characteristic

Under these assumptions, the switching behaviors are given by the following equations:

i) During $t_i > t \geq 0$, if it is assumed that $v = 0$ at $t = 0$,

$$C\frac{dv}{dt} + f(v) = I_0 + I_i - \frac{v}{Z_0}\,,\quad \overrightarrow{V} = v,\quad \overleftarrow{V} = 0 \tag{70}$$

where $\overrightarrow{V}$ and $\overleftarrow{V}$ are voltages of the forward and backward traveling wave of the transmission line at $x = 0$, and the characteristic impedance of the transmission line is Z_0.

ii) During $2t_0 > t \geq t_i$,

$$C\frac{dv}{dt} + f(v) = I_0 - \frac{v}{Z_0}\,,\quad \overrightarrow{V} = v,\quad \overleftarrow{V} = 0 \tag{71}$$

iii) During $t \geq 2t_0$,

$$\left.\begin{aligned} C\frac{dv}{dt} + f(v) &= I_0 - \frac{v}{Z_0} + \frac{2}{Z_0}\left(\frac{R-Z_0}{R+Z_0}\right)\overrightarrow{V}(t-2t_0) \\ \overrightarrow{V}(t) = v(t) - \overleftarrow{V}(t) &= v(t) - \frac{R - Z_0}{R + Z_0}\overrightarrow{V}(t-2t_0) \\ \overleftarrow{V}(t) &= \frac{R - Z_0}{R + Z_0}\overrightarrow{V}(t-2t_0) = \rho\overrightarrow{V}(t-2t_0) \end{aligned}\right\} \tag{72}$$

where it is assumed that the transmission line is ideal. Even if the transmission line has some attenuation, but no phase distortion, the attenuation can be included in the reflection coefficient.

These equations, with the assumption of Figure 13, become in the region of $E > v > 0$:

i) During $t_r > t \geq 0$

$$\frac{dv}{d\tau} + (1-GZ_0)v = (I_0+I_i-I)Z_0 \tag{73}$$

where $\tau = t/CZ_0$. If it is assumed that $v = 0$ at $t = 0$ and $I_0 + I_i + I > 0$

$$v = \frac{JZ_0}{a}(1-\epsilon^{-a\tau}) \tag{74}$$

where $a = 1 - GZ_0$, $J = I_0 + I_i - I > 0$. J should be positive for triggering. The rise time t_r may be written as

$$\tau_r = \frac{1}{a} \ell n \frac{JZ_0}{JZ_0 - aE} \tag{75}$$

And for $a = 0$

$$\tau_r = \frac{E}{JZ_0} \tag{76}$$

The necessary and sufficient condition for complete switching is

$$JZ_0 > aE \text{ and } J > 0 \tag{77}$$

because it is assumed that the rise time t_r is shorter than the input pulse width t_i. The output voltage is kept at E after $t = t_r$.

ii) During $2t_0 > t \geq t_i$

$$\frac{dv}{d\tau} + av = (I_0 - I)Z_0 \tag{78}$$

a) If $(I_0-I)Z_0 - aE = I_0Z_0 - E \geq 0$, the output voltage stays at E.

b) If $Z_0I_0 - E < 0$, the circuit flies back, that is,

$$v = \left(\frac{Z_0I_0-E}{a}\right)\left[1 - \epsilon^{-a(\tau-\tau_i)}\right] \tag{79}$$

where $\tau_i = t_i/CZ_0$.

iii) During $t \geq 2t_0$

a) In the case of $Z_0I_0 - E \geq 0$, the value of the output voltage is kept at E until $t = t' + 2t_0$ which is given by

$$Z_0I_0 - E = \frac{-2\rho JZ_0}{a}\left[1 - \epsilon^{-a\tau'}\right] \tag{80}$$

During $t_i + 2t_0 > t \geq t' + 2t_0$

$$\frac{dv}{d\tau} + av = Z_0(I_0-I) + \frac{2\rho JZ_0}{a}\left[1 - \epsilon^{-a(\tau-2\tau_0)}\right] \tag{81}$$

Therefore

$$av = \left(\frac{Z_0I_0-E+2\rho JZ_0}{a}\right)\left[1 - \left\{1 + a(\tau-\tau'-2\tau_0)\right\}\epsilon^{-a(\tau-\tau'-2\tau_0)}\right] + aE \tag{82}$$

If, in the above equation, v is less than zero at $t = t_r + 2t_0$, the fall time $t_f = CZ_0$ of the output voltage becomes

$$\left(\frac{Z_0I_0-E+2\rho JZ_0}{a}\right)\left[1-(1+a\tau_f)\epsilon^{-a\tau_f}\right] + aE = 0 \tag{83}$$

The output voltage is usually kept at zero after $t = t_f + t' + 2t_0$. If v exceeds zero by v_0 at $t = t_r + 2t_0$, then

$$\frac{dv}{d\tau} + av = Z_0(I_0-I) + 2\rho E \tag{84}$$

and

$$av = \left\{(I_0-I)Z_0 + 2\rho E - av_0\right\}(1-\epsilon^{-a(\tau-\tau_r-2\tau_0)}) + av_0 \tag{85}$$

In this case, the fall time may be written as

$$\tau_f = -\frac{1}{a}\,\ell n\left|\frac{Z_0(I_0-I_1)+2\rho E}{Z_0(I_0-I_1)+2\rho E - av_0}\right| + \tau_r \tag{86}$$

b). For $Z_0I_0 - E < 0$, Eq. (81) is applied during $t_r + 2t_0 > t \geq 2t_0$ with the initial condition that

$$v\,|_{\tau=2\tau_0} = \frac{Z_0I_0 - E}{a}\left[1 - \epsilon^{-a(2\tau_0-\tau_i)}\right] + E \tag{87}$$

The procedure after that is the same as for a). The results of the rise time calculations t_r, the dead time t' and the fall time t_f of the output waveforms are shown in Figure 14, where I = 1a, E = 1v, G = $1(\Omega)^{-1}$, $I_0 = 0.8a$, $I_i = 0.4a$, $\rho = -1$ and Z_0 is changed from 1Ω to 3.75Ω. From this calculation, it is concluded that the best value of the characteristic impedance of the transmission line, where the rise time becomes equal to the fall time, is approximately 2.8Ω. In order that the circuit is switched off by the backward wave of the transmission line, the characteristic impedance of the transmission line should be kept between 1.25Ω and 3.75Ω.

If it is less than 1.25Ω, the circuit flies back by itself after termination of the triggering pulse. If it is larger than 3.75Ω, it does not fly back in this ideal case. The waveforms are shown in Figure 15.

5. Binary Counter

The circuit which is composed of a tunnel diode and an open-ended transmission line as shown in Figure 16(a) has the characteristic of a binary counter for a bipolar signal which is shown in Figure 16(b). The most important merits of this counter are its simplicity and very high speed behavior. The demerits are the necessity of an uncommon input signal and the complicated interstage circuitry.

It is assumed that each input signal consists of a positive pulse followed by a delayed negative pulse. (A negative pulse and a delayed positive pulse can also be considered.) When the tunnel diode is in the low voltage state, the first positive pulse sets the tunnel diode into the high voltage state and sends a positive forward wave to the transmission line. The delayed negative input pulse would reset the tunnel diode to the low voltage state, but the positive backward wave of a transmission line generated by a positive input pulse cancels the negative input pulse and inhibits the device from returning to the low voltage state, that is, the input signal sets the circuit which is in a low voltage state to a high voltage state. When the tunnel diode is in a high voltage state, the first positive input pulse does not change the state of the tunnel diode and sends only small positive forward waves into the transmission line. But now the negative pulse of the input signal resets the circuit to a low voltage state.

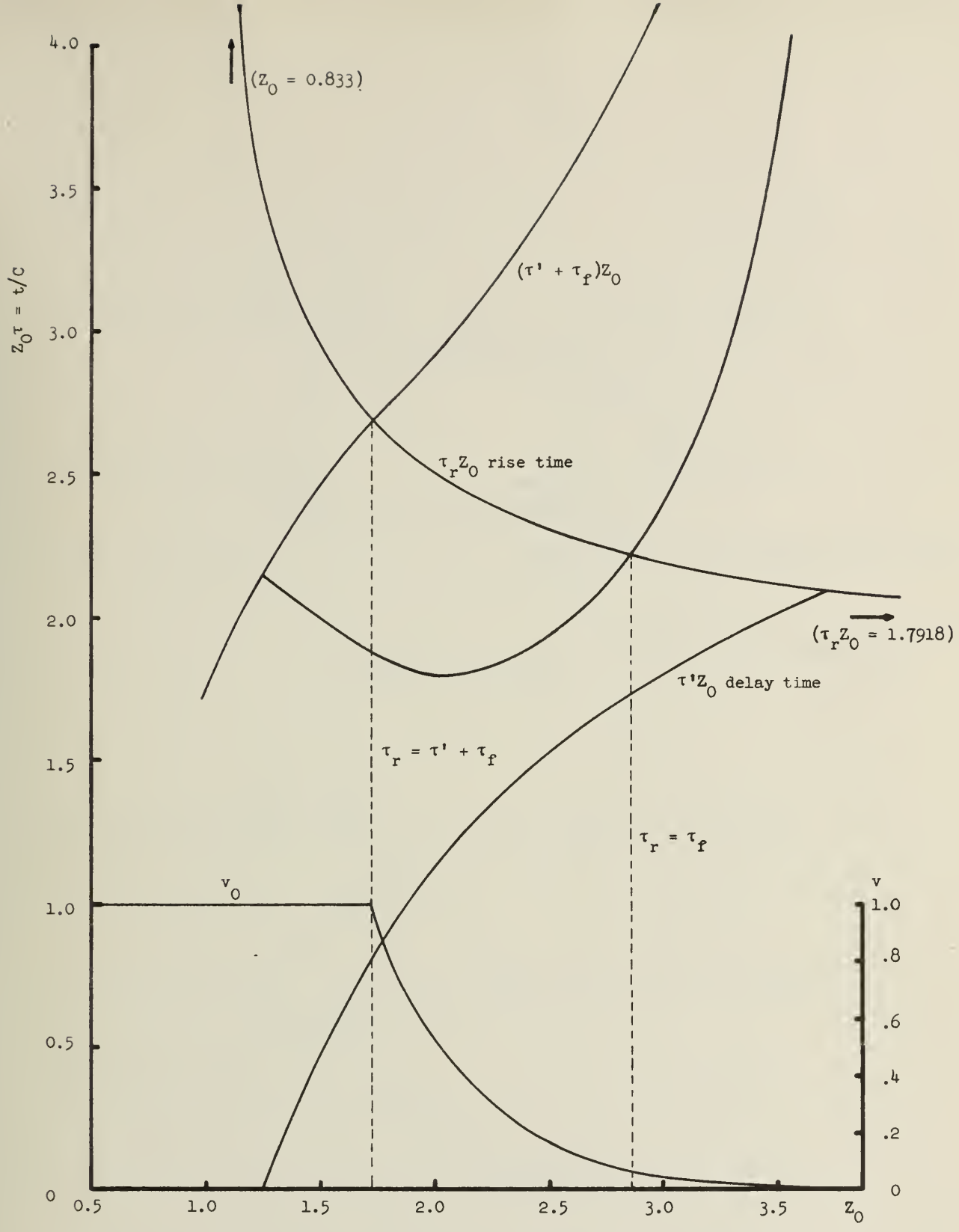
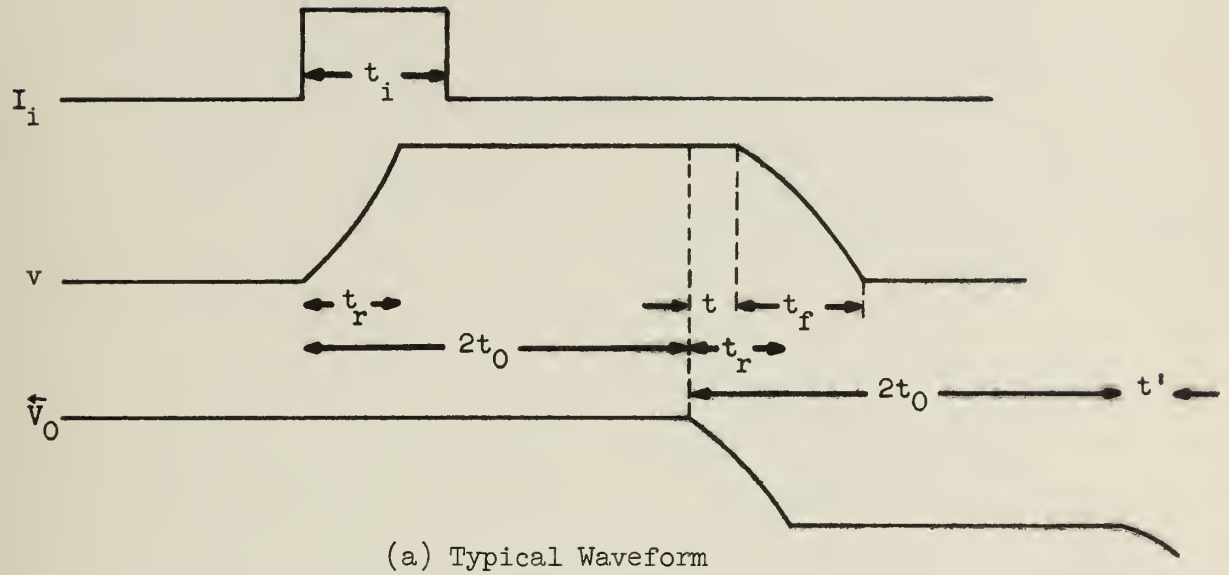
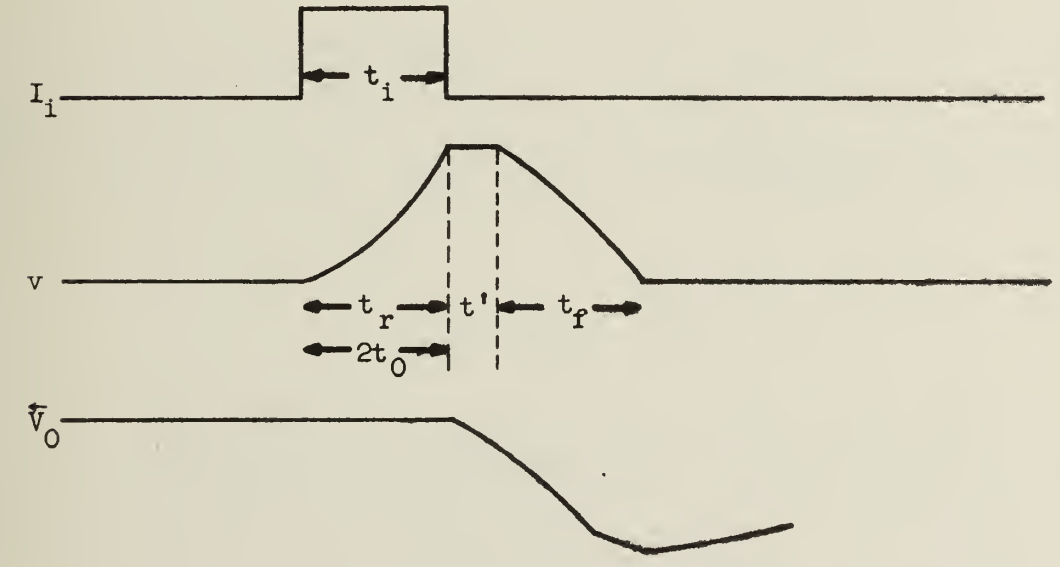


Figure 14. τ_r, τ' and τ_f of the Monostable Multivibrator

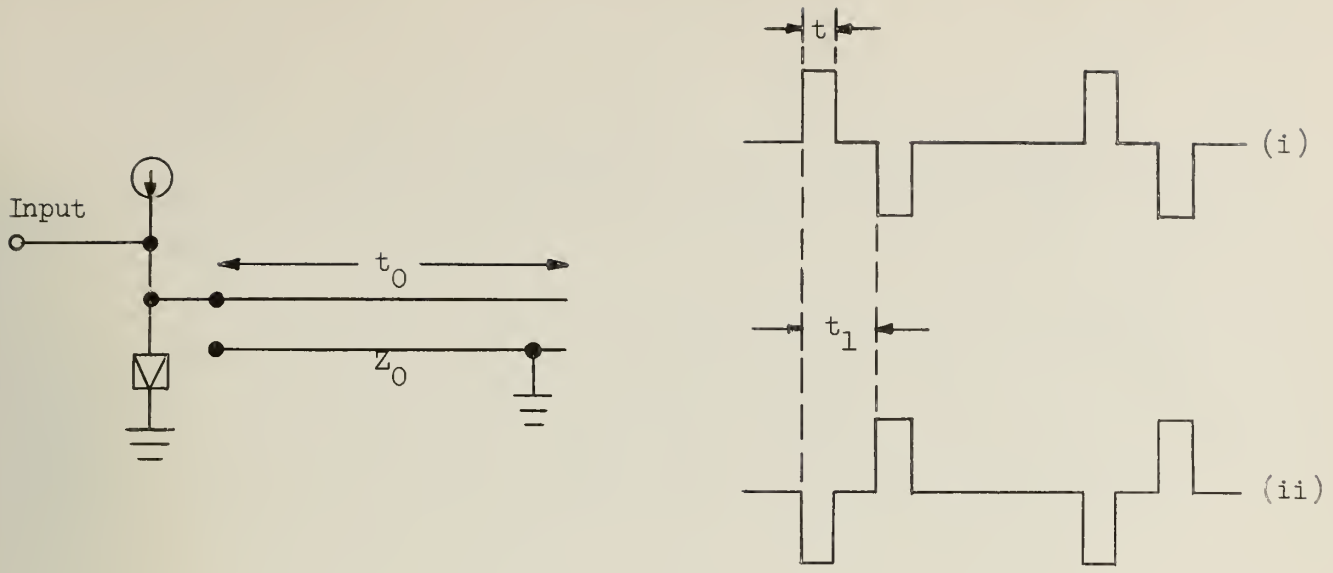


(a) Typical Waveform



(b) Ideal Waveform $t_r = 2t_0 = t_1 = t_f$

Figure 15. Waveforms of the Monostable Multivibrator



(a) Counter Circuit (b) Input Waveforms

Figure 16. Tunnel Diode Counter

This circuit can be analyzed in nearly the same way as the monostable multivibrator. The delay time of the second input pulse (t_d) should be longer than twice the transmission time of the transmission line ($2t_0$) plus the switching time (t_s), but it should be shorter than four times the transmission time, that is,

$$2t_0 + t_s \leq t_d < 4t_0$$

Of course, it is preferable that the input pulse width is equal to or longer than the switching time;

$$t_i \geq t_s$$

The maximum repetition frequency of the input signals depends on the attenuation factor of the transmission line and the reflection coefficient at the tunnel diode side, and so on.

6. Conclusion

As the results of the analyses of the typical tunnel diode circuits coupled into transmission lines, the many design conditions of these circuits became evident.

It is difficult to derive the characteristics of the combined circuits of these standard circuits directly from these results. As the next stage, it is necessary to investigate the interconnecting circuits. These circuits should have the characteristic of directivity and they should also have the characteristic of the input-output separation in order to make the designs of systems easy.

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